DIFFRACTION OF LIGHT

INTRODUCTION

When light from a narrow linear slit is incident on the sharp edge of an obstacle, it will be found that there is illumination to some extent within the geometrical shadow of the obstacle. This shows that light can bend round an obstacle. All phenomena like this which are produced when the incident wavefront is somehow limited are called diffraction of light.

When waves encounter obstacles (or openings), they bend round the edges of the obstacles, if the dimensions of the obstacles are comparable to the wavelength of the waves. The bending of waves around the edges of an obstacle is called diffraction.

Diffraction of Light: If an opaque obstacle (or aperture) be placed between a source of light and a screen, a sufficiently distinct shadow (or an illuminated region) is obtained on the screen. This shows that light travels approximately in straight lines. If, however, the size of the obstacle or the aperture is small (comparable to the wavelength of light), then there is a departure from straight-line propagation, and the light bends round the corners of the obstacle or the aperture, and enters the geometrical shadow. This bending of light is called 'diffraction.' As a result of diffraction, the edges of the shadow (or illuminated region) are not sharp, but the intensity is distributed in a certain way depending upon the nature of the obstacle or the aperture.

Radio waves diffract around building, but not light waves. For noticeable diffraction of waves, their wavelength should be of the order of the size of the obstacle. The radio waves have wavelengths of the order of the size of the walls and windows of buildings (= 10^{-1} — 10^{4} m) and so they are easily diffracted. On the other hand, the wavelength of light waves (= 10^{-6} m) is too small to be diffracted around buildings.

FRESNEL AND FRAUNHOFFER TYPES OF DIFFRACTION:

The diffraction phenomena are broadly classified into two types: Fresnel diffraction and Fraunhoffer diffraction.

1. **Fresnel diffraction**: In this type of diffraction, the source of light and the screen are effectively at finite distances from the obstacle (Fig. a). Observation of Fresnel diffraction phenomenon does not require any lenses. The incident wave front is not planar. As a result, the phase of secondary wavelets is not the same at all points in the plane of the obstacle. The resultant amplitude at any point of the screen is obtained by the mutual interference of secondary wavelets from different elements of unblocked portions of wave front. It is experimentally simple but the analysis proves to be very complex.

2. **Fraunhoffer diffraction**: In this type of diffraction, the source of light and the screen are effectively at infinite distances from the obstacle. Fraunhoffer diffraction pattern can be easily observed in practice. The conditions required for Fraunhoffer diffraction are achieved by using two convex lenses, one to make the light from the source parallel and the other to focus the light after diffraction on to the screen (Fig. b). The diffraction is thus produced due to interfere between parallel rays. The incident wave front as such is plane and the secondary

wavelets which originates from the unblocked portions of the wave front, are in the same phase at every point in the obstacle. This problem is simple to handle mathematically because the rays are incoming light is rendered parallel with a lens and diffracted beam is focused on another lens.

The above distinction creates an impression that a plane wavefront is essential for Fraunhofer diffraction. This is, however, not necessary. Fraunhoffer diffraction pattern may be obtained even with spherical or cylindrical wavefronts. As a matter of fact, in Fraunhofer's diffraction, the diffraction pattern is an image of the source modified by diffraction at the diffracting obstacle or the aperture. (In Fresnel's diffraction, the pattern is a shadow of the diffracting obstacle or the aperture modified, by diffraction effects). Hence the essential requirement for the Fraunhofer's diffraction is that the pattern must be observed in the plane which is conjugate to the plane in which the source of light lies. Fraunhofer's diffraction is a limiting case of the more general Fresnel's diffraction.

Difference between Interference and Diffraction

In the phenomenon of interference, the interference occurs between waves starting from two (or more, but finite in number of coherent sources. In diffraction, on the other hand, the interference occurs between the secondary wavelets starting from different points, infinite in number, of the same wave. However, both are superposition effects and often both are present simultaneously, as in Young's experiment. The interference and diffraction patterns differ in the following respects:

(I) In an interference pattern the minima are usually almost perfectly dark while in a diffraction pattern they are not so.

(ii) In an interference pattern all the maxima are of same intensity but not so in the diffraction pattern.

(iii) The interference fringes are usually equally-spaced (although not always). The diffraction fringes are never equally-spaced.

The effect is found to be significant when the dimension of the diffracting element becomes comparable with the wavelength of light.

Fresnel gave a satisfactory explanation of this phenomenon by using Huygen's Principle in conjunction with the principle of superposition. According to Huygens's Principle each point on the wavefront acts as a source of secondary wave. The mutual interference of these secondary waves derived from a particular wavefront, produces the phenomenon of diffraction. Thus interference effect is due to the superposition of two distinct waves coming from two coherent sources while diffraction is the effect of superposition of the secondary waves coming from the different parts of the same wavefront.

All optical instruments use only a limited portion of the incident wavefront and hence some diffraction effects are always present in the image. Diffraction effects are accordingly of great importance in the detailed understanding of optical devices.

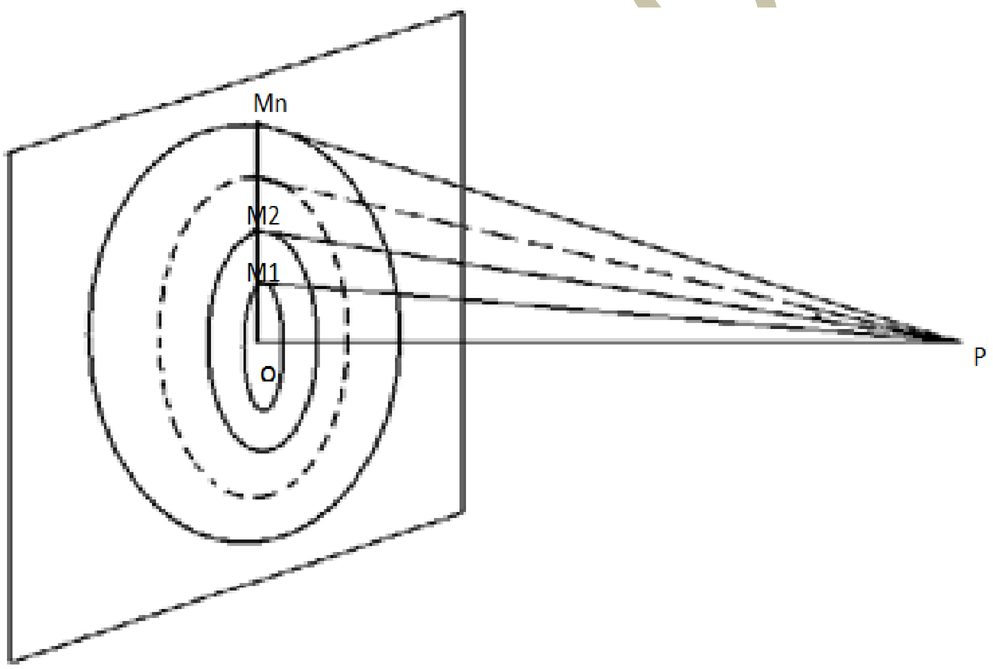
FRESNEL'S HALF PERIOD ZONES OR STRIPS

To obtain the intensity at a point due to a wavefront, Fresnel divide the wavefront into few concentric circular zones or strips in such a way that the light coming from the successive half period zones or strips meet at the point in opposite phase i.e. phase difference of $\lambda/2$. These zones or strips are called Half Period Zones or Strips.

******* *The lights coming from successive zones or strips meet at the point in opposite phase. This phase difference is equal to the change in phase difference in half of a time period (T). That is why it is called Half Period Zones or Strips.**************

CONSTRUCTION:

Let ABCD be the plane wavefront. P is an external point at a distance 'b' perpendicular to the wavefront. To find the resultant intensity at 'P' due to the whole wavefront, few concentric spheres are drawn taking 'P' as centre and $b, b+\lambda/2, b+2.\lambda/2$............... etc as radii (λ is wavelength of the light). These spheres cut out the wavefront into concentric circular areas of radii OM_1, OM_2, OM_3............ on the wavefront. The central circular region is called first half period zone. The region between the first and the second circular region is called second half period zone and so on.



RADIUS OF HALF PERIOD ZONE

Radius of the nth half period zone,

$$OM_n = [M_nP^2 - OP^2]^{1/2}$$

$$= [(b + \frac{n\lambda}{2})^2 - b^2]^{1/2}$$

$$n = 1, 2, 3, ...$$

$$= [b^2 + bn\lambda + \frac{n^2\lambda^2}{4} - b^2]$$

$$\approx (bn\lambda)^{1/2}$$

$$\approx \sqrt{bn\lambda}$$

Thus the radii of half period zones are proportional to roots of natural numbers. In other words, radii of first, second, third etc half period zones have radii in ratio $\sqrt{1}$, $\sqrt{2}$, $\sqrt{3}$ etc.

AREA OF HALF PERIOD ZONES:

Area of the nth half period zone is

$$A_n = \pi[OM_n{}^2 - OM_{n-1}{}^2]$$

$$= \pi[nb\lambda - (n-1)b\lambda] \quad \text{[Radius of nth zone } \approx \sqrt{bn\lambda}\text{]}$$

$$= \pi b\lambda$$

Thus the area of the half period zone is constant and does not depend on the order of the zone. Each zone has equal area. The area is

i) directly proportional to wavelength (λ)

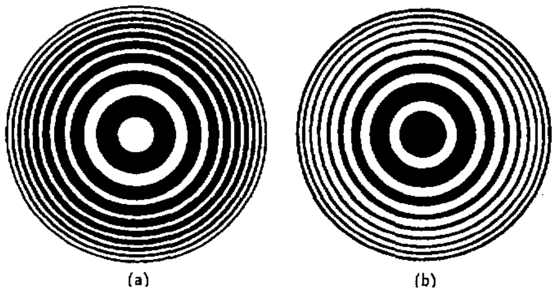
ii) directly proportional to distance (b) of the wavefront from the point (P)

ZONE PLATE

A zone plate is specially designed optical device based on the Fresnel theory of half period zone i.e. constructing concentric circular zones such that light from alternate zones are cut off. Zone plate has a converging property.

CONSTRUCTION:

To construct a zone plate concentric circles of radii proportional to integral no. are drawn in a white paper. The odd numbered zones are blackened and a reduced photograph is taken. In developed negative the odd zones become transparent to incident light and even numbered zones will cut the light off.



Uses: Zone plate has lensing properties and can be used as converging lens. The correctness of Fresnel's method of dividing the wavefront into half period zones can be verified with the help of a zone plate.

THEORY:

The light from successive half period zones of a wavefront meet at a point in opposite phases. Thus the resultant intensity at the point due to the whole wavefront reduces and is equal to one fourth of intensity due to the first half period zone. If the alternate (odd or even numbered) zones are made opaque, then light coming from the transparent zones meet at the same phase and resultant intensity increases.

RESULTANT AMPLITUDE AT A POINT DUE TO THE WHOLE WAVEFRONT:

Though the area of each zone is same, the obliquity factor increases with increase in order no. of the zones. Thus amplitude at point 'P' decreases gradually.Let $a_1, a_2, a_3......$ etc are amplitude at 'P' due to first, second, third etc. Half period zones, then $a_1 > a_2 > a_3......$ since each zone is out of phase by phase π to its neighbouring zone, the resultant amplitude at 'P' due to all zones at any instant

$$R = a_1 - a_2 + a_3 - a_4 + ...(-1)^{n-1} a_n$$

We can write, $a_2 = \frac{a_1 + a_3}{2} \quad a_4 = \frac{a_3 + a_5}{2} \quad a_6 = \frac{a_5 + a_7}{2}$ etc.

$$R = \frac{a_1}{2} + [\frac{a_1}{2} - a_2 + \frac{a_3}{2}] + [\frac{a_3}{2} - a_4 + \frac{a_5}{2}] + [\frac{a_5}{2} - a_6 + \frac{a_7}{2}] + ...$$

$$i.e. R = \frac{a_1}{2} + \frac{a_n}{2} \quad \text{if n is odd}$$

$$or R = \frac{a_1}{2} + \frac{a_{n-1}}{2} - a_n \quad \text{if n is even}$$

For large value of n, a_n, a_{n-1} tends to 0.

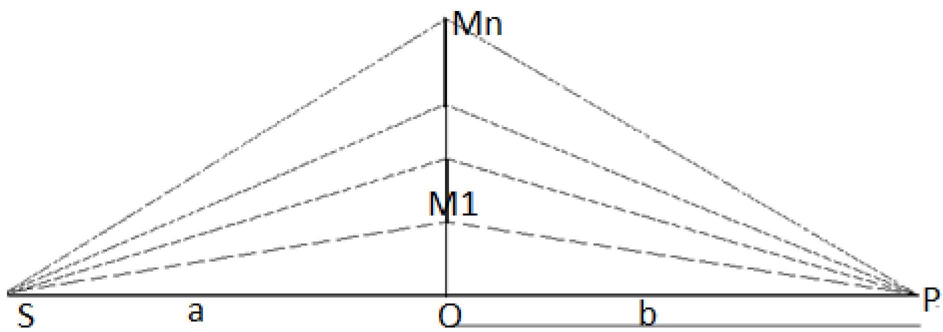
Therefore, resultant amplitude at ‘P’

$$R = \frac{a_1}{2}$$

The resultant intensity $I \alpha \frac{a_1^2}{4}$

i.e. resultant intensity due to whole wavefront is only one fourth that due to the first zone.

LENSING PROPERTY OF ZONE PLATE:



Let XY be a section of zone plate where OM_1, OM_2, OM_3............ etc are equal to radii r_1, r_2, r_3............. etc of the first, second, third etc half period zones. S is source of light at distance ‘a’ and ‘P’ is position of the bright image at distance ‘b’. The position of the plate is such that path difference increases from one zone to next zone by $\frac{\lambda}{2}$.

$$SO + OP = a + b$$

$$SM_1 + M_1P = a + b + \frac{\lambda}{2}$$

$$SM_2 + M_2P = a + b + 2.\frac{\lambda}{2} etc$$

$$SM_1 = (SO^2 + OM_1{}^2)^{1/2} = (a^2 + r_1{}^2)^{1/2}$$

$$M_1P = (OP^2 + OM_1{}^2)^{1/2} = (b^2 + r_1{}^2)^{1/2}$$

$$\therefore (1) \Rightarrow \; (a^2 + r_1^2)^{1/2} + (b^2 + r_1^2)^{1/2} = a + b + \frac{\lambda}{2}$$

$$\Rightarrow \; a(1 + \frac{r_1^2}{a^2})^{1/2} + b(1 + \frac{r_1^2}{b^2})^{1/2} = a + b + \frac{\lambda}{2}$$

$$\Rightarrow \; a + \frac{r_1^2}{2a} + b + \frac{r_1^2}{2b} = a + b + \frac{\lambda}{2}$$

$$\Rightarrow \; \frac{r_1^2}{2}(\frac{1}{a} + \frac{1}{b}) = \frac{\lambda}{2}$$

$$\Rightarrow \; r_1^2(\frac{1}{a} + \frac{1}{b}) = \lambda$$

Similarly, for nth zone

$$\Rightarrow \; r_n^2(\frac{1}{a} + \frac{1}{b}) = n\lambda$$

$$\Rightarrow \; \frac{1}{a} + \frac{1}{b} = \frac{n\lambda}{r_n^2}$$

Applying sign convention

$$\frac{1}{b} - \frac{1}{a} = \frac{n\lambda}{r_n^2} = \frac{1}{f_n}$$

where

$$f_n = \frac{r_n^2}{n\lambda}$$

Eqn (ii) is equivalent to lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ with ‘a’ and ‘b’ as object distance and image distance. Thus the zone plate acts as a converging lens. A zone plate has a no. of foci which depend on the numbers of zone used as well as wavelength of light used.

Multiple foci of zone plate:

Zone plate has multiple foci depending on the numbers of zone used. The bright focus is at the distance $f_1 = \frac{r_n^2}{n\lambda}$

As the point of observation is approached, the area of zone diminishes. When the point is at distance $f_2 = \frac{r_n^2}{3n\lambda} = \frac{f_1}{5}$, the first transparent zone of the plate contains first, second and third zone. Thus brighter image is formed with lesser intensity. For point at distance $f_3 = \frac{r_n^2}{5n\lambda} = \frac{f_1}{5}$, the clear zone contains first, second, third, fourth and fifth zone resulting bright image at the points. Thus numbers of foci are obtained.

Differences between convex lenses and zone plate:

1. For a given wavelength of light, a convex lens has one focus whereas a zone plate possesses multiple foci. The brightest image is obtained at $f_1 = \frac{r_n^2}{n\lambda}$

r_n is the radius of n-th half period zone. Other bright images with decreasing intensity are obtained at $f_3 = \frac{r_n^2}{3n\lambda}$ $f_5 = \frac{r_n^2}{5n\lambda}$
,

2. Light from the consecutive transparent zones of the zone plate arrives at the image point after one complete period of wave, whereas in case of lens light reaches the image point in the same phase.

3. The focal length of the zone plate decreases as wavelength increases but in case of lens focal length increases with increase in wave length. When white light is used red image is formed nearer to the zone plate but away from the lens.

4. Thickness of the lens is not uniform but thickness of the zone plate is uniform.

FRESNEL DIFFRACTION

Diffraction at a straight edge:

Let A be straight edge of an opaque obstacle ‘AB’. It is illuminated by monochromatic light of wavelength λ coming from source S. The diffraction

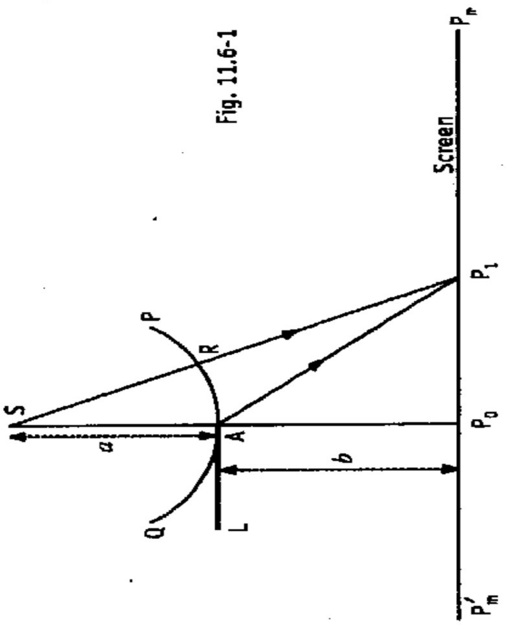


Fig. 11.6-1

pattern is observed at screen

Explanation:

Let PAQ be cylindrical wavefront from linear source ‘S’. With respect to point ‘P_1’, R is the pole of the wavefront. Similarly ‘A’ is the pole of the wavefront w.r.t. ‘P_o’. Both halves of the wavefront are divided into numbers of half period elements.

For the point ’P_o’, the upper half AP of the wavefront is exposed while lower half ‘AB’ is completely obstructed. The resultant amplitude is therefore $a_1/2$, where a_1 is amplitude due to central element. As one goes below P_o, the pole of the wavefront shifts towards Q and gradually, the first, the first two, the first three half period elements of upper half along with the complete lower half are obstructed. Therefore, at different points below ‘Po’ the resultant displacements are $-a_2/2$, $a_3/2$, $-a_4/2$...etc. since $a_4<a_3<a_2<a_1$, the intensity rapidly decreases.

On the other hand, as one moves above P_o, the pole shifts towards ‘P’ and one after another, the first, the first two, the first three etc. Half period elements of lower half will be exposed along with the complete upper half. The illumination at a point P_1 depends on whether the no. of exposed element on lower half is odd or even. If the no. is odd, the point will be bright or if even it is dark. Thus alternate dark and bright fringes are observed above 'P1'

The no of half period elements in 'AR' depends on path difference AP_1-RP_1

Let, $SA = a$ and $AP_o = b$ and $P_oP_1 = x_m$

Then $SP_1^2 = (a+b)^2 + x_m{}^2$

$$\Rightarrow SP_1 = (a+b)[1+\frac{x_m{}^2}{(a+b)^2}]^{1/2}$$

$$= (a+b)[1+\frac{1}{2}\frac{x_m{}^2}{(a+b)^2}]$$

$$= (a+b)+\frac{1}{2}\frac{x_m{}^2}{(a+b)}$$

Therefore, $RP_1 = SP_1 - SR$

$$= (a+b)+\frac{1}{2}\frac{x_m{}^2}{(a+b)} - a$$

$$= b+\frac{x_m{}^2}{2(a+b)}$$

Similarly, $AP_1 = b+\frac{x_m{}^2}{2b}$

$$AP_1 - RP_1 = x_m{}^2[\frac{1}{2b}-\frac{1}{2(a+b)}] = \frac{x_m{}^2 a}{2b(a+b)}$$

For maximum intensity at P_1,

$$\frac{x_m{}^2 a}{2b(a+b)} = (2m+1)\frac{\lambda}{2}$$

$$\Rightarrow x_m = \sqrt{\frac{(2m+1)b(a+b)\lambda}{a}}$$

For minimum intensity at P_1,

$$\frac{x_m{}^2 a}{2b(a+b)} = m\lambda$$

$$\Rightarrow x_m = \sqrt{\frac{(2b(a+b)m\lambda}{a}}$$

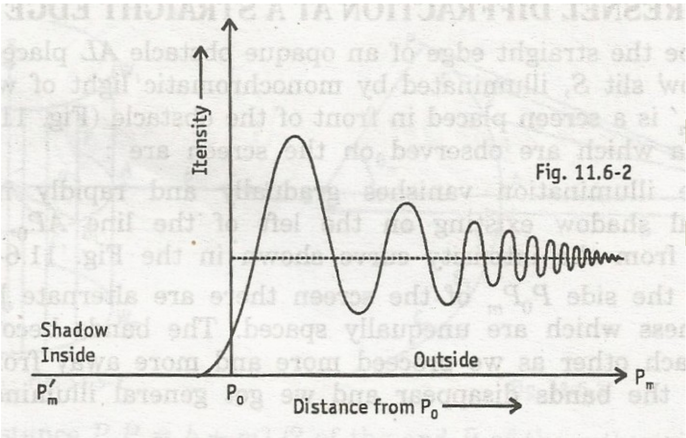
$$x_{m+1}^{2} - x_m^{2} = 2k \quad \text{where} \quad k = \frac{b(a+b)\lambda}{a}$$

$$\Rightarrow x_{m+1} - x_m = \frac{2k}{x_{m+1} + x_m}$$

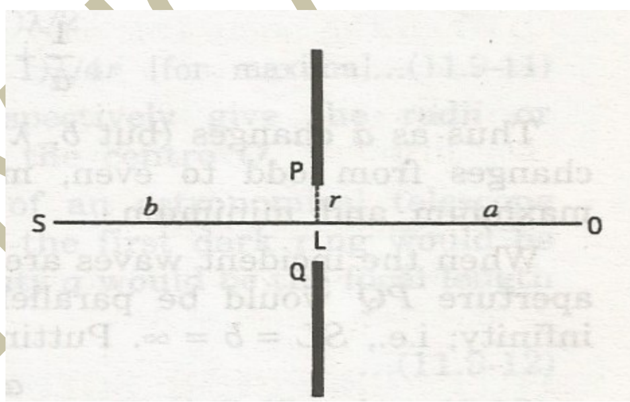
$$\approx \frac{k}{x_m}$$

This gives the expression for m-th fringe width.

The intensity distribution in case of diffraction by straight edge is shown in fig.



Diffraction at circular aperture:



Let PQ is the small circular aperture. S is a point source and O is a point on the screen on the axial line SLO. MN is the geometrical image of PQ on the screen.

The intensity at ‘O’ depends on the number of half period zones contained by the aperture w.r.t. to ‘O’. If there is even no. of zones then they mutually cancel each other in pairs and intensity will be less. On the other hand if the aperture contains odd no. of zones, intensity at ‘O’ will be maximum due to the unpaired zone. If the screen is moved towards the aperture, the distance ‘LO’ changes and the aperture contains alternately odd and even number of zones with respect to ‘O’ and hence the point ‘O’ will be alternately dark and bright.

Intensity at an axial point:

Let,

$$OL = a$$
$$SL = b$$
$$PL = r$$

By geometry

$$OP = (a^2 + r^2)^{1/2} = a + \frac{1}{2}\frac{r^2}{a}$$

$$SP = (b^2 + r^2)^{1/2} = b + \frac{1}{2}\frac{r^2}{b}$$

Path difference between marginal and central rays is

$$l = (SP + PO) - (SL + LO) = m\frac{\lambda}{2}$$

$$\Rightarrow (a + \frac{1}{2}\frac{r^2}{a} + b + \frac{1}{2}\frac{r^2}{b}) - (a + b) = m\frac{\lambda}{2}$$

$$\Rightarrow \frac{r^2}{2}(\frac{1}{a} + \frac{1}{b}) = m\frac{\lambda}{2}$$

$$\Rightarrow (\frac{1}{a} + \frac{1}{b}) = m\frac{\lambda}{r^2}$$

Keeping b, λ and r constant when 'a' is changed, 'm' becomes alternately odd and even and the point 'O' will be alternately bright and dark.

For parallel rays $b = \alpha$

$$\therefore a = \frac{r^2}{m\lambda}$$

i.e. the first bright and the second bright will be obtained at $a_1 = \frac{r^2}{\lambda}$ and $a_2 = \frac{r^2}{3\lambda}$ respectively.

Therefore separation between two successive bright spot is

$$a_1 - a_2 = \frac{r^2}{\lambda} - \frac{r^2}{3\lambda} = \frac{2r^2}{3\lambda}$$

Similarly first and the second dark are formed at

$$a_1^{/} = \frac{r^2}{2\lambda} - \frac{r^2}{4\lambda} = \frac{r^2}{4\lambda}$$

Intensity at a point within geometrical image

The intensity at a point ‘R_1’ within geometrical image except axial point becomes maximum and minimum according as odd or even number of half period zones are remaining in each half of the exposed wave front above and below the new pole.

Intensity at a point beyond geometrical image

Let ‘R’ be a point beyond geometrical image and at a distance ‘ x_m ’ from ‘O’

Path difference of ‘R’ from ‘P’ and ‘Q’

$$\begin{aligned} l &= QR - PR \\ &= \sqrt{a^2 + (x_m + r)^2} - \sqrt{a^2 + (x_m - r)^2} \\ &= a + \frac{1}{2}\frac{(x_m + r)^2}{a} - a - \frac{1}{2}\frac{(x_m - r)^2}{a} \\ &= \frac{2rx_m}{a} \end{aligned}$$

For minimum intensity at ‘R’

$$\frac{2rx_m}{a} = m\frac{\lambda}{2}$$

$$\Rightarrow x_m = \frac{am\lambda}{2r}$$

And for maximum intensity

$$\frac{2rx_m}{a} = (2m+1)\frac{\lambda}{2}$$

$$\Rightarrow x_m = \frac{(2m+1)a\lambda}{4r}$$

Eqn (6) and (7) respectively give the distances or radii of dark and bright rings from the centre ‘O’.

The objective of a telescope consists of an achromatic convex lens and a circular aperture is fixed in front of the lens. Let the diameter of the aperture be (D=2r). While viewing distant objects, the incident wavefront is plane and the diffraction pattern consists of a bright centre surrounded by dark and bright rings of gradually decreasing intensity. The radii of the dark rings are given by

$$x_m = \frac{am\lambda}{2r} = \frac{am\lambda}{D}$$

The radius of the first dark ring is, $x_1 = \dfrac{a\lambda}{D}$

For an incident plane wavefront, a =f , the focal length of the objective.

Therefore, $$x_1 = \frac{f\lambda}{D}$$

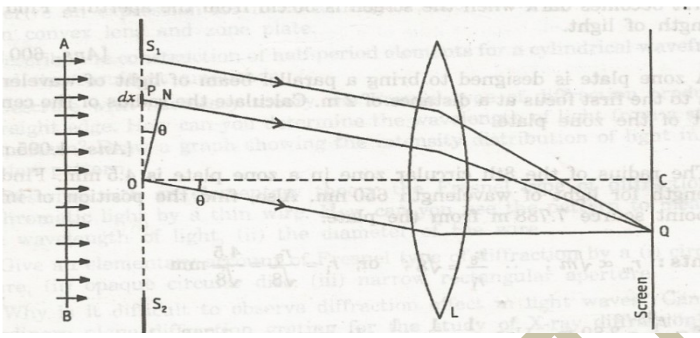
x_1 gives the distance of the first secondary minimum from the central bright maximum. However, according to Airy's theory, the radius of the first dark ring is given by

$$x_1 = \frac{1.22 f\lambda}{D}$$

it is interesting to note that the size of the central image depends on λ, the wavelength of light, f focal length of the lens and , D diameter of the lens aperture.

FRAUNHOFER DIFFRACTION IN A SINGLE SLIT

Let parallel beam of monochromatic light of wavelength λ be incident on a slit S_1S_2 normally. According to Huygen's theory every point on the incident wavefront behaves like an independent source of secondary wavelets. These wavelets are focused on the screen by convex lens L.



The beams normal to the slit are focussed at point 'C' on the screen, producing a bright fringe there. The beams diffracted at angle 'Θ' meet the screen at point 'Q'.

Intensity at point Q:

The phase difference between the light coming from midpoint 'O' of the slit and at a point 'P', x distance apart from 'O' is

$$\frac{2\pi}{\lambda}.PN = \frac{2\pi}{\lambda}x\sin\Theta = lx$$

where, $l = \frac{2\pi}{\lambda}\sin\Theta$

If disturbance at 'O' is considered as $Ae^{i\omega t}$, then disturbance at 'Q' due to the waves from P is proportional to $e^{i(\omega t - lx)}$

The disturbance at 'Q' due to the diffracting element 'dx' can be written as

$$dy = CAe^{i(\omega t - lx)}dx$$

The disturbance at 'Q' due to the whole slit, $y = \int_{-a/2}^{+a/2} CAe^{i(\omega t - lx)}dx$

$$= CAe^{i\omega t}.a\frac{\sin(la/2)}{(la/2)}$$

$$= CAa\frac{\sin(la/2)}{(la/2)}e^{i\omega t}$$

The resultant intensity at 'Q', $I = y.y^* = (CAa)^2 \frac{\sin^2(la/2)}{(la/2)^2}$

$$= I_o \frac{\sin^2\alpha}{\alpha^2}$$

where $I_o = (CAa)^2$ = maximum intensity and

$$\alpha = \frac{la}{2} = \frac{\pi}{\lambda} a\sin\Theta$$

Thus intensity depends on α which in turn depends on angle of diffraction 'Θ'.

For maximum and minimum intensity, $\frac{dI}{d\alpha} = 0$

$$\Rightarrow \sin\alpha(\alpha\cos\alpha - \sin\alpha) = 0$$

i.e. either $\sin\alpha = 0$

or $\sin\alpha = \alpha\cos\alpha$

$$\Rightarrow \tan\alpha = \alpha$$

a) For $\sin\alpha = 0$, $\frac{d^2I}{d\alpha^2} = +ve$

therefore the condition for minimum intensity is $\sin\alpha = 0$

$$\Rightarrow \sin\alpha = \sin m\pi \text{ where, } m = \pm 1, \pm 2, \pm 3......$$

$$\therefore \alpha = m\pi$$

$$\Rightarrow a\sin\Theta = m\lambda$$

$m = \pm 1, \pm 2, \pm 3......$ gives the 1st, 2nd, 3rd etc. minima on either sides of the central maximum.

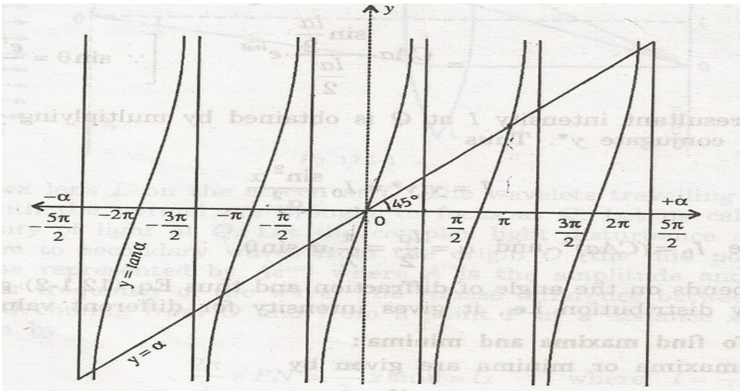
(b) for $m = 0$, $\alpha = 0$

Since $\lim_{\alpha \to 0} \frac{\sin^2\alpha}{\alpha^2} = 1$, intensity will be maximum (I_o) i.e. for $a\sin\alpha = 0$, the central bright fringe is obtained.

(c) for $\tan\alpha = \alpha$, $\frac{d^2I}{d\alpha^2} = -ve$

$\therefore$ Condition for maxima is $\tan\alpha = \alpha$

The values of α corresponding to the points of intersection of graphs $y=\pm\alpha$ and $y=\tan\alpha$ satisfy the eqn. The values are obtained as $\alpha=0, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}$........



$$\Rightarrow a\sin\Theta=(2m+1)\lambda/2, \quad m=\pm1,\pm2,\pm3......$$

The exact values of α are 0, 1.430π, 2.462π, 3.471π ....

$\alpha=0$ represents principal maximum whereas other values give the secondary maxima.

Intensity:

Intensity of principal maximum is

$$I=\lim_{\alpha\to 0}\frac{\sin^2\alpha}{\alpha^2}=I_o$$

Intensity of secondary maximum is

$$I_1=\lim_{\alpha\to 0}\frac{\sin^2(\frac{3\pi}{2})}{(\frac{3\pi}{2})^2}=\frac{4}{9\pi^2}I_o\approx\frac{I_o}{22}$$

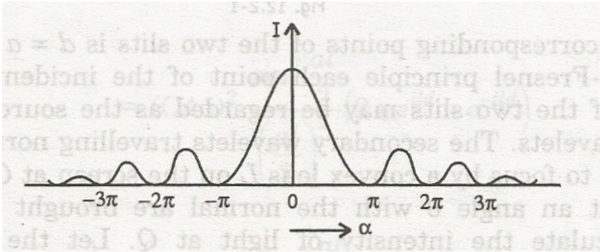
Intensity of second secondary maximum is

$$I_2=\lim_{\alpha\to 0}\frac{\sin^2(\frac{5\pi}{2})}{(\frac{5\pi}{2})^2}=\frac{4}{25\pi^2}I_o\approx\frac{I_o}{61}$$

and so on.

Thus the secondary maxima are very feeble and their intensities fall rapidly with increase in order no.

The intensity distribution is shown in fig.



Thus the diffraction pattern consists of bright principal maximum in the direction of the incident light having alternately minima lie at $\alpha = \pm\pi, \pm 2\pi, \pm 3\pi.........$. The subsidiary maxima do not fall exactly midway between two minima but are displaced towards the centre of the pattern by an amount which decreases with increasing order.

WIDTH OF CENTRAL MAXIMUM:

The angle of diffraction Θ_1 for first minimum on either sides of central maximum is given as

$$a \sin\Theta_1 = \lambda$$

$$\Theta_1 \approx \sin\Theta_1 = \frac{\lambda}{a}$$

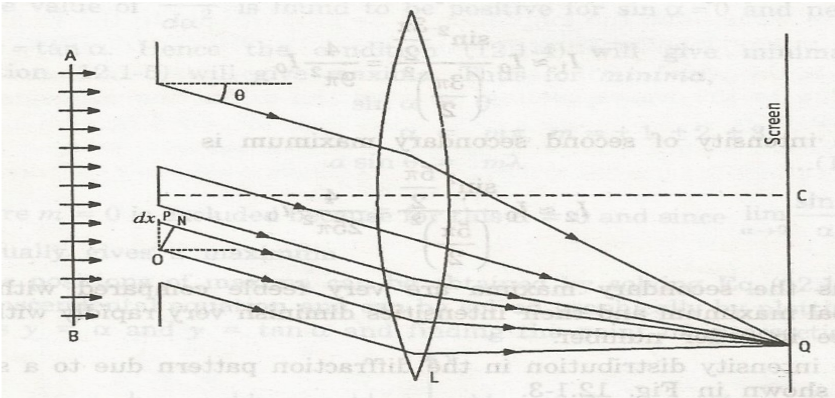
$\therefore$ Angular with of central maximum is $2\Theta_1 = \frac{2\lambda}{a}$ which is inversely proportional to slitwidth(a).

What happens if the slit is narrower?

Angular with of central maximum is $2\Theta_1 = \frac{2\lambda}{a}$ which is inversely proportional to slit width (a). When the slit width is narrowed a decreases and the central maximum becomes wider. When the slit width is as small as the wavelength ($a = \lambda$) then the first minimum occurs at $\theta = 90^o$, which means the central maximum fills the whole space.

FROUNHOFER DIFFRACTION IN A DOUBLE SLIT

Let parallel beam of monochromatic light of wavelength λ be incident on a double slit normally. Let a be width of each slit and b be separation between them. The distance between any pair of corresponding points of the two slits is $d = a + b$ According to Huygen's theory every point on the incident wavefront behaves like an independent source of secondary wavelets. These wavelets are focused on the screen by convex lens L.



The beams normal to the slit are focussed at point 'C' on the screen, producing a bright fringe there. The beams diffracted at angle 'Θ' meet the screen at point 'Q'.

Intensity at point Q:

The phase difference between the light coming from midpoint 'O' of the slit and at a point 'P', x distance apart from 'O' is

$$\frac{2\pi}{\lambda}.PN = \frac{2\pi}{\lambda} x \sin\Theta = lx$$

where, $l = \dfrac{2\pi}{\lambda}\sin\Theta$

If disturbance at 'O' is considered as $Ae^{i\omega t}$, then disturbance at 'Q' due to the waves from P is proportional to $e^{i(\omega t - lx)}$

The disturbance at 'Q' due to the diffracting element 'dx' can be written as

$$dy = CAe^{i(\omega t - lx)} dx$$

The disturbance at 'Q' due to the double slit

$$y = \int_{-a/2}^{+a/2} CAe^{i(\omega t - lx)} dx + \int_{d-a/2}^{d+a/2} CAe^{i(\omega t - lx)} dx$$

$$= CAe^{i\omega t}\left[\frac{e^{-ilx}}{-il}\right]_{-a/2}^{+a/2} + CAe^{i\omega t}\left[\frac{e^{-ilx}}{-il}\right]_{d-a/2}^{d+a/2}$$

$$=CAa\frac{\sin(la/2)}{(la/2)}[1+e^{-ild}]e^{i\omega t}$$

The resultant intensity at ‘Q’

$$I = y.y^*$$

$$=(CAa)^2\frac{\sin^2(la/2)}{(la/2)^2}[1+e^{-ild}][1+e^{ild}]$$

$$=(CAa)^2\frac{\sin^2(la/2)}{(la/2)^2}[2+e^{ild}+e^{-ild}]$$

$$=(CAa)^2\frac{\sin^2(la/2)}{(la/2)^2}[2+2\cos ld]$$

$$=4I_o\frac{\sin^2\alpha}{\alpha^2}\cos^2\beta$$

.... (6)

where $I_o=(CAa)^2$, $\alpha=\frac{la}{2}=\frac{\pi}{\lambda}a\sin\Theta$ and $\beta=\frac{ld}{2}=\frac{\pi}{\lambda}(a+b)\sin\Theta$

Thus the resultant intensity depends on two factors

i) $I_o\frac{\sin^2\alpha}{\alpha^2}$,which gives diffraction pattern due to a single slit

ii) $\cos^2\beta$, which gives interference pattern due to the diffracted light beams from the two slits.

Condition for minima:

The resultant intensity would be zero when either factor in eqn (6) is zero.

a) the factor $\frac{\sin^2\alpha}{\alpha^2}=0$ $\sin\alpha=0$ but $\alpha\neq 0$. Therefore, $\alpha=m\pi$ $m=\pm1,\pm2,\pm3......$

Or $a\sin\Theta=m\lambda$

These minima are called diffraction minima.

b) Interference minima are obtained for $\cos^2\beta=0$

i.e. $\beta=\pm(2s+1)\pi/2$; $s=1,2,3............$

or $(a+b)\sin\Theta=\pm(2s+1)\lambda/2$

these minima are known as the interference minima.

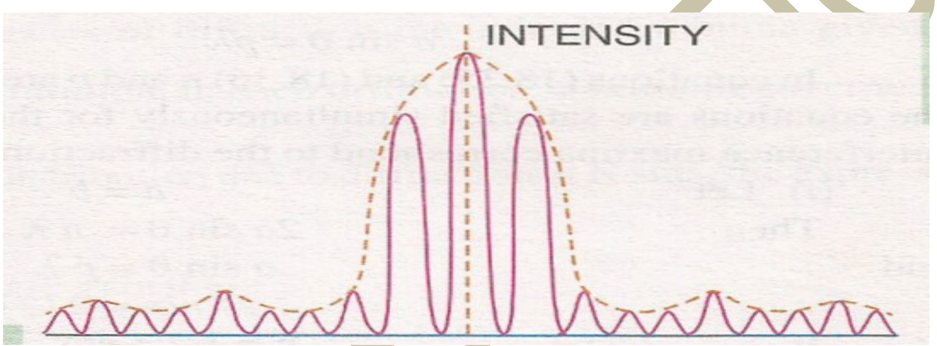
Condition for maxima:

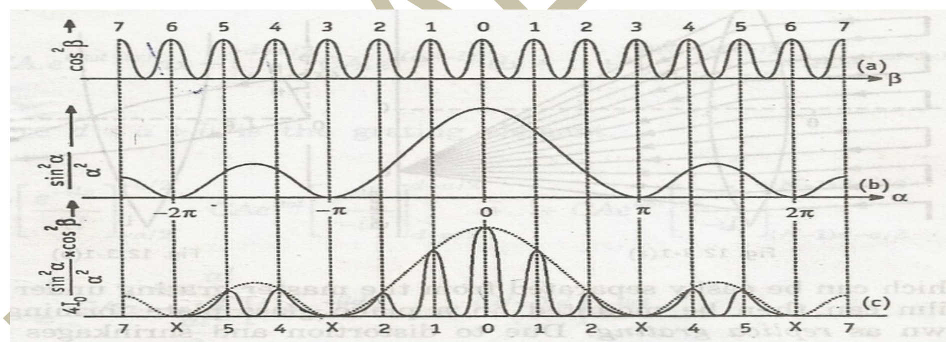
c) Diffraction maxima due to the factor $\frac{\sin^2\alpha}{\alpha^2}$ are obtained at $\alpha = 0, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}$........

d) the interference maxima due to the factor $\cos^2\beta$ are obtained for $\beta = p\pi$, $p = 0, \pm1, \pm2, \pm3$......

i.e. $(a+b)\sin\Theta = p\lambda$

The intensity distribution is shown in fig.





Missing order:

If the condition for maxima of interference pattern and minima of diffraction pattern are simultaneously satisfied for a given value of Θ then the corresponding interference maxima will be missing. In that case

$$\frac{a+b}{a} = \frac{p}{m}$$

Thus for missing order the ratio $\frac{a+b}{a}$ should be the ratio of two integers.

For example, if $\frac{a+b}{a}=\frac{2}{1}$ *; or* $a=b$ *then* $p=2m$ *. Hence 2, 4, 6 etc orders of interference maxima are absent which corresponds to 1, 2, 3, etc orders of diffraction dark bands. There will be 3 interference maxima (corresponding to p=0,* ± 1*) in the central diffraction maximum.*

Effect of increasing the slit-width: If we increase the slit-width the envelope of the fringe-pattern changes so that its central peak is sharper. The fringe-spacing, which depends on slit-separation, does not change. Hence interference maxima now fall within the central diffraction maximum.

Effect of increasing the distance between slits: If the slit-width 'a' is kept constant and the separation 'b' between them is increased, the fringes become closer together, the envelope of the pattern remaining unchanged. Thus more interference maxima fall within the central envelope.

Effect of increasing wavelength: On increasing λ, the envelope becomes broader, and the fringes move further apart.

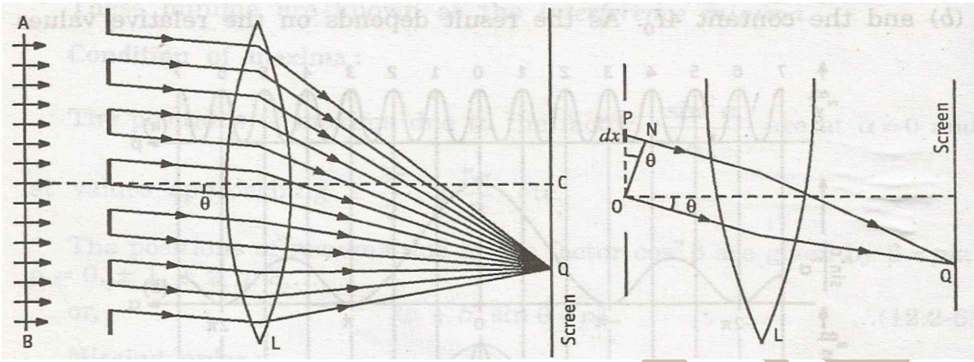
DISTINCTION BETWEEN SINGLE SLIT AND DOUBLE SLIT DIFFRACTION PATTERNS

The single slit diffraction pattern consists of a central bright maximum with secondary maxima and minima of gradually decreasing intensity. The double slit diffraction pattern consists of equally spaced interference maxima and minima with in the central maximum. The intensity of the central maximum in diffraction pattern due to a double slit is four times that of the central maximum in the diffraction pattern due to diffraction at a single slit. In the above arrangement, if one of the slits is covered with opaque screen, the pattern observed is similar to the one observed with a single slit.

The spacing of diffraction maxima and minima depends on 'a', the width of the slit and the spacing of the interference maxima and minima depends on the value of 'a' and 'b' where 'b' is opaque spacing between the two slits. The intensities of the interference maxima are not constant and decrease to zero on either side of the central maximum. These maxima reappear two or three times before the intensity becomes too low to be observed.

DIFFRACTION IN PLANE TRANSMISSION GRATING:

A diffraction grating is an arrangement equivalent to a large number of parallel slits of equal widths and separated from one another by equal opaque spaces. It is made by ruling a large number of fine, equidistant and parallel lines on an optically-plane glass plate with a diamond point. The ruling scattered the light and are effectively opaque while the unrolled pars transmit light and acts as slits.



Let parallel beam of monochromatic light of wavelength λ be incident on a plane transmission grating normally. Let a be width of each slit and b be width of each opaque space between the slits. The distance between any pair of corresponding points of the two slits is $d = a + b$ which is called grating element or grating constant According to Huygen's theory every point on the incident wavefront behaves like an independent source of secondary wavelets. These wavelets are focused on the screen by convex lens L.

The beams normal to the slit are focussed at point 'C' on the screen, producing a bright fringe there. The beams diffracted at angle 'Θ' meet the screen at point 'Q'.

Intensity at point Q:

The phase difference between the light coming from midpoint 'O' of the slit and at a point 'P', x distance apart from 'O' is

$$\frac{2\pi}{\lambda}.PN = \frac{2\pi}{\lambda} x \sin\Theta = lx$$

$$\frac{2\pi}{\lambda}.PN = \frac{2\pi}{\lambda} x \sin\Theta = lx$$

where, $l = \frac{2\pi}{\lambda}\sin\Theta$

If disturbance at 'O' is considered as $Ae^{i\omega t}$, then disturbance at 'Q' due to the waves from P is proportional to $e^{i(\omega t - lx)}$

The disturbance at 'Q' due to the diffracting element 'dx' can be written as

$$dy = CAe^{i(\omega t - lx)}dx$$

The disturbance at 'Q' due to the double slit

$$y = \int_{-a/2}^{+a/2} CAe^{i(\omega t - lx)}dx + \int_{d-a/2}^{d+a/2} CAe^{i(\omega t - lx)}dx + \int_{2d-a/2}^{2d+a/2} CAe^{i(\omega t - lx)}dx .... + \int_{(N-1)d-a/2}^{(N-1)d+a/2} CAe^{i(\omega t - lx)}dx$$

$$= CAe^{i\omega t}\left[\frac{e^{-ilx}}{-il}\right]_{-a/2}^{+a/2} + CAe^{i\omega t}\left[\frac{e^{-ilx}}{-il}\right]_{d-a/2}^{d+a/2} + CAe^{i\omega t}\left[\frac{e^{-ilx}}{-il}\right]_{2d-a/2}^{2d+a/2} .... + CAe^{i\omega t}\left[\frac{e^{-ilx}}{-il}\right]_{(N-1)d-a/2}^{(N-1)d+a/2}$$

$$= CAa\frac{\sin(la/2)}{(la/2)}[1 + e^{-ild} + ... + e^{-i(N-1)ld}]e^{i\omega t}$$

$$= CAa\frac{\sin(la/2)}{(la/2)}\frac{e^{-iNld} - 1}{e^{-ild} - 1}.e^{i\omega t}$$

The resultant intensity at 'Q'

$$I = y.y^*$$

$$= (CAa)^2\frac{\sin^2(la/2)}{(la/2)^2}\frac{e^{-iNld} - 1}{e^{-ild} - 1}.\frac{e^{+iNld} - 1}{e^{+ild} - 1}$$

$$= (CAa)^2\frac{\sin^2(la/2)}{(la/2)^2}\frac{2 - e^{+iNld} - e^{-iNld}}{2 - e^{+ild} - e^{-ild}}$$

$$= I_o\frac{\sin^2\alpha}{\alpha^2}\frac{\sin^2 N\beta}{\sin^2\beta}$$

Where $I_o = (CAa)^2$, $\alpha = \frac{la}{2} = \frac{\pi}{\lambda}a\sin\Theta$ and $\beta = \frac{ld}{2} = \frac{\pi}{\lambda}(a+b)\sin\Theta$

Thus the resultant intensity depends on two factors

i) $I_o\frac{\sin^2\alpha}{\alpha^2} = I_1$,which gives diffraction pattern due to a single slit

ii) $\frac{\sin^2 N\beta}{\sin^2\beta} = I_2$, which gives interference pattern due to the diffracted light beams from all the slits.

PRINCIPAL MAXIMA:

If the slit width 'a' is very small and observation is confined to the neighbourhood of the central pattern the variation of the factor $I_o \frac{\sin^2 \alpha}{\alpha^2} = I_1$ is small and under this condition the maxima will be solely controlled by the factor $\frac{\sin^2 N\beta}{\sin^2 \beta} = I_2$

This factor is maximum when $\beta = m\pi$; $m = 0, \pm 1, \pm 2, \pm 3......$

or $(a+b)\sin\Theta = m\lambda$

For $\beta = m\pi$, $I_2 = \frac{0}{0}$ and hence indeterminate. But in limit $\beta \to m\pi$ we get the maximum value of I_2.

$$\lim_{\beta \to m\pi} \frac{\sin N\beta}{\sin \beta} = \lim_{\beta \to m\pi} \frac{N \cos N\beta}{\cos \beta} = N$$

This gives $I_2 = N^2$ and $I = I_{pm} = I_0 \frac{\sin^2 \alpha}{\alpha^2} \times N^2 = I_1 \times N^2$

Thus the intensity of the principal maxima increases as the number of slits (N) increases, but due to the presence of the factor $\frac{\sin^2 \alpha}{\alpha^2}$, whose value decreases with increase of the angle of diffraction (Θ), the intensity of principal maxima decreases with the increase in the order number of bands.

Condition for secondary minima and maxima:

The factor $\frac{\sin^2 N\beta}{\sin^2 \beta} = I_2$ depends on β and for maxima or minima we must have $\frac{dI_2}{d\beta} = 0$

Now $\frac{dI_2}{d\beta} = \frac{2N \sin N\beta . \cos N\beta}{\sin^2 \beta} - \frac{2 \sin^2 N\beta . \cos \beta}{\sin^3 \beta}$

$$= 2 \frac{\sin^2 N\beta}{\sin^2 \beta} (N \cot N\beta - \cot \beta)$$

Hence for maxima or minima, either $\frac{\sin N\beta}{\sin \beta} = 0$ or $N \cot N\beta = \cot \beta$

a) Secondary minima:

When $\sin N\beta = 0$ but $\sin\beta \neq 0$ then $\frac{\sin N\beta}{\sin\beta} = 0$ and hence intensity becomes zero (i.e. minimum). Thus for minimum $N\beta = \pm s\pi$

or $(a+b)\sin\Theta = \pm\frac{s}{N}\lambda$

where s has integral values excepting $0, N, 2N, 3N$ etc. as for these values of 's', $\sin\beta = 0$ and we obtained principal maxima. Thus it is evident from equ and equ that between two consecutive principal maxima there are $(N-1)$ minima. Hence there will be $(N-2)$ other maxima known as secondary maxima between any two adjacent principal maxima.

b) Secondary maxima

The condition $N\cot N\beta = \cot\beta$ makes $\frac{dI_2}{d\beta} = 0$. Also, it can be showed that this condition makes $\frac{d^2I_2}{d\beta^2}$ negative. Thus the value of β which satisfy the condition $N\cot N\beta = \cot\beta$ will give the positions of secondary maxima, excepting $\beta = m\pi$ which gives principal maxima.

Intensity of secondary maxima:

For secondary maxima, $N\cot N\beta = \cot\beta$

Or
$$N^2\frac{\cos^2 N\beta}{\cos^2\beta} = \frac{\sin^2 N\beta}{\sin^2\beta}$$

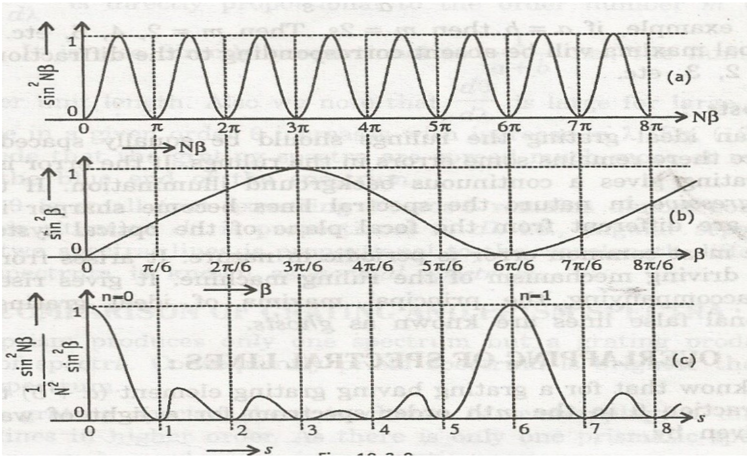
Or
$$N^2\frac{(1-\sin^2 N\beta)}{1-\sin^2\beta}$$

Or
$$N^2\frac{(1-\sin^2 N\beta)}{1-\sin^2\beta} = \frac{N^2\sin^2 N\beta}{N^2\sin^2\beta} = \frac{N^2}{1+(N^2-1)\sin^2\beta}$$

Hence the intensity of the secondary maxima

$$\frac{I_{sm}}{I_{pm}} = \frac{1}{1+(N^2-1)\sin^2\beta}$$

This equation shows that as N increases the intensity of secondary maxima relative to the principal maxima decreases. When N is very large the secondary maxima becomes very weak. This is why secondary maxima are not generally observed with a grating having large N.



ABSENT SPECTRA:

For the m th order principal maximum in the direction Θ, we have the condition

$$(a+b)\sin\Theta = m\lambda$$

Suppose that the value of a is such that s th order diffraction minimum occurs in the same direction then

$$a\sin\Theta = s\lambda$$

If these two conditions are satisfied simultaneously then m th order principal maximum will be absent from the resulting spectra.

From the above eqns

$$\frac{a+b}{a} = \frac{m}{s}$$

GHOST LINES:

In an ideal grating the rulings should be equally spaced. But in practice there remain some errors in the rulings. If the error is random the grating gives a continuous background illumination. If the error is progressive in nature the spectral lines become sharper in planes which are different from the focal plane of the optical system.The most common error is periodic in nature. It arises from defects in the driving mechanism of the ruling machine. It gives rise to false lines accompanying the principal maxima of ideal grating. These additional false lines are known as ghost lines.

***Reference book: A Text Book on Light by B. Ghosh & K.G. Mazumdar*